1 Kinematics

1.1 Equations of motion

For objects in uniformly accelerated rectilinear motion, the five quantities, displacement x, time taken t, initial velocity v_0 , final velocity v and acceleration a are related by a set of simple equations called kinematic equations of motion:

$$v = v_o + at$$

$$x = v_o t + \frac{1}{2}at^2$$

$$v^2 = v_o^2 + 2ax$$

if the position of the object at time $t = 0$ is 0. If the particle starts at $x = x_0$, x in above equations is replaced by $(x – x_0)$

1.2 Projectile motion

An object that is in flight after being projected is called a projectile. If an object is projected with initial velocity v_o making an angle $_o$ with x-axis and if we assume its initial position to coincide with the origin of the coordinate system, then the position and velocity of the projectile at time t are given by:

$$x = (v_o \cos\theta_o)t$$

$$y = (v_o \sin\theta_o)t - \frac{1}{2}gt^2$$

$$v_x = v_{ox} = v_o \cos\theta_o$$

$$v_y = v_o \sin\theta_o - gt$$

The path of a projectile is parabolic and is given by:

$$y = (\tan\theta_o)x - \frac{gx^2}{2(v_o \cos\theta_o)^2}$$

The maximum height that a projectile attains is:

$$h_m = \frac{(v_o \sin\theta_o)^2}{2g}$$

The time taken to reach this height is:

$$t_m = \frac{v_o \sin\theta_o}{g}$$

The horizontal distance travelled by a projectile from its initial position to the position it passes $y = 0$ during its fall is called the range, R of the projectile. It is:

$$R = \frac{v_o^2}{g}\sin 2\theta_o$$

2 Laws of Motion

Newton's second law of motion:
The rate of change of momentum of a body is proportional to the applied force and takes place in the direction in which the force acts. Thus

$$\vec{F} = k\frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = k\ m\ \vec{a}$$

where $\vec{F}$ is the net external force on the body and its acceleration.
We set the constant of proportionality $k = 1$ in SI units. Then

$$\vec{F} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = m\vec{a}$$

2.1 Circular Motion

2.1.1 Motion of a car on a level road

$$v_{max} = \sqrt{\mu R g}$$

2.1.2 Motion of a car on a banked road

$$v_{max} = \left(Rg\frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}\right)^{\dfrac{1}{2}}$$

For $\mu = 0$,

$$v_o = (Rg\tan\theta)^{\frac{1}{2}}$$

3 Work, energy and Power

The work-energy theorem states that the change in kinetic energy of a body is the work done by the net force on the body

$$K_f - K_i = W_{net}$$

The gravitational potential energy of a particle of mass m at a height x about the earth's surface is

$$v(x) = mgx$$

where the variation of g with height is ignored. The elastic potential energy of a spring of force constant k and extension x is

$$V(x) = \frac{1}{2}kx^2$$

4 System of particles and Rotational Motion

The centre of mass of a system of n particles is defined as the point whose position vector is

$$R = \frac{\sum m_i \vec{r}_i}{M}$$

The angular momentum of a system of n particles about the origin is

$$\vec{L} = \sum_{i=1}^{n} \vec{r}_i \times \vec{p}_i$$

The torque or moment of force on a system of n particles about the origin is

$$\tau = \sum_{1} \vec{r}_i \times \vec{F}_i$$

The force F_i acting on the i^th particle includes the external as well as internal forces.Assuming Newton's third law of motion and that forces between any two particles act along the line joining the particles, we can show $\tau_{int} = 0$ and

$$\frac{\mathrm{d}\vec{L}}{\mathrm{d}t} = \tau_{ext}$$

The moment of inertia of a rigid body about an axis is defined by the formula

$$I = \sum m_i r_i^2$$

where r_i is the perpendicular distance of the i^th point of the body from the axis. The kinetic energy of rotation is $K = \frac{1}{2}I^2$ For rolling motion without slipping $v_cm = R\omega$, where v_cm is the velocity of translation (i.e. of the centre of mass), R is the radius and m is the mass of the body. The kinetic energy of such a rolling body is the sum of kinetic energies of translation and rotation:

$$K = \frac{1}{2}mv_c^2m + \frac{1}{2}I\omega^2$$

5 Gravitation

Newton's law of universal gravitation states that the gravitational force of attraction between any two particles of masses m_1 and m_2 separated by a distance r has the magnitude

$$F = g\frac{m_1 m_2}{r^2}$$

where G is the universal gravitational constant, which has the value $6.67210^{-11} Nm^2kg^{-2}$ g–2 Kepler's third law states that The square of the orbital period of a planet is proportional to the cube of the semi major axis of the elliptical orbit of the planet The period T and radius R of the circular orbit of a planet about the Sun are related by

$$T^2 = \left(\frac{4\pi^2}{GM_s}\right) R^3$$

where M_s is the mass of the Sun. Most planets have nearly circular orbits about the Sun. For elliptical orbits, the above equation is valid if R is replaced by the semi-majoraxis, a. The acceleration due to gravity:

5.1 at a height h above the earth's surface

$$g(h) = \frac{GM_E}{(R_E + h)^2}$$

5.2 at depth d below the surface of the Earth

$$g(d) = \frac{GM_E}{R_E^2 \left(1 - \frac{d}{R_E}\right)} = g(0)$$

The gravitational force is a conservative force, and therefore a potential energy function can be defined. The gravitational potential energy associated with two particles separated by a distance r is given by

$$V = -\frac{Gm_1m_2}{r}$$

where V is taken to be zero at $r \to \infty$. If an isolated system consists of a particle of mass m moving with a speed v in the vicinity of a massive body of mass M, the total mechanical energy of the particle is given by

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

That is, the total mechanical energy is the sum of the kinetic and potential energies. The total energy is a constant of motion. The escape speed from the surface of the earth is

$$v_e = \sqrt{\frac{2GM_E}{R_E}} = 2gR_E$$

and has a value of $11.2kms–^{1\cdot}$